Numerical versus optical layer oriented: a comparison in terms of SNR

Dolores Bello\textsuperscript{1}, Jean-Marc Conan\textsuperscript{1}, Gerard Rousset\textsuperscript{1}, Massimiliano Tordi\textsuperscript{2}, Roberto Ragazzoni\textsuperscript{2}, Elise Vernet-Viard\textsuperscript{2}, Markus Kasper\textsuperscript{3} and Stefan Hippler\textsuperscript{4}

\textsuperscript{1} Office National d'Etudes et de Recherches Aerospatiales (ONERA), Chatillon cedex, France
\textsuperscript{2} Osservatorio Astrofisico di Arcetri (OAA), Firenze, Italy
\textsuperscript{3} European Southern Observatory (ESO), Garching bei Munchen, Germany
\textsuperscript{4} Max-Planck Institute f"{u}r Astronomie (MPIA), Heidelberg, Germany

ABSTRACT

Multiconjugate adaptive optics employing several deformable mirrors conjugated at different altitudes has been proposed in order to extend the size of the corrected field of view [FOV] with respect to the size of the corrected FOV given by a classical adaptive optics system. A three dimensional measurement of the turbulent volume is needed in order to collect the information to command the several deformable mirrors. Given a set of guide stars in the field of view, this can be done both using tomography, in which several wavefront sensors are used, each of them coupled to one of the guide stars, or layer oriented techniques, in which wavefront sensors are coupled to a given layer in the atmosphere, and collect light from the whole set of guide stars. We will call this type of measurements optical layer oriented. This type of measurements can be also obtained combining in a numerical way, tomographic measurements. This hybrid approach is called numerical layer oriented. In order to compare their performance, we present an analytical study of the signal to noise ratio [SNR] in the measurements for the two techniques. Optical layer oriented is shown to be more efficient in the range of faint flux and large number of guide stars, while low detector noise will allow numerical layer oriented schemes to be more efficient in terms of SNR.

Keywords: Multiconjugate adaptive optics, wavefront sensing, layer oriented.

1. INTRODUCTION

Adaptive optics systems are now working in a large number of telescopes\textsuperscript{1-3} allowing to obtain diffraction limited images. However, due to the vertical distribution of the atmospheric turbulence, the size of the corrected FOV is limited to a the isoplanatic angle (typically, a few arcsecs). In order to extend the compensated FOV, the concept of multiconjugate adaptive optics has been proposed\textsuperscript{4,5}. This technique uses several deformable mirrors conjugated at different altitudes. These deformable mirrors are controlled using the information about the phase measured using several guide stars. A tomographic approach was first proposed\textsuperscript{6} in which a wavefront sensor is coupled with each guide star and then all the information is processed and employed to control each deformable mirror (see Figure 1(a)). In the recent past years the layer oriented concept has been proposed by R. Ragazzoni\textsuperscript{7,8} (see Figure 1(b)). It is based on measuring the phase with a single wavefront sensor with one or more wavefront sensing planes which can be optically conjugated to given layers in the atmosphere. A deformable mirror [DM] is associated with each of the wavefront sensing planes, and the wavefront sensor collects light from the whole set of guide stars. This technique was first conceived with the idea of making this kind of measurements in an optical way. However, measurements of the same nature can be reproduced combining numerically tomographic measurements where each wavefront sensor is coupled to a single guide star.

Further author information: (Send correspondence to D. Bello)

Dolores Bello: E-mail: cbello@ill.iac.es / bello@onera.fr; URL:http://www.onera.fr; Phone +33146 734757; Fax +33146 734171

Jean-Marc Conan: E-mail: conan@onera.fr; URL:http://www.onera.fr; Phone +33146 734748; Fax +33146 734171
Section 2 explains in detail the differences between optical and numerical layer oriented measurements. Analytical formulas for the SNR corresponding to each technique are developed in Section 3, while Section 4 is devoted to the comparison of the SNR of the two techniques for several study cases. Finally, Section 5 gives a discussion of advantages and disadvantages of each technique from other considerations than SNR.

2. OPTICAL AND NUMERICAL LAYER ORIENTED MEASUREMENTS

Figure 1 shows the schemas for the tomographic and layer oriented approach. In the tomographic approach each wavefront sensor is coupled to a single guide star. The information from all the wavefront sensors is combined and processed in the wavefront controller and allows to obtain the commands for the deformable mirrors of the system. Each wavefront sensor collects information from the column of turbulence along the direction of the guide star it is coupled to. The sampling of the volume of turbulence using several analysis directions allows to reconstruct the turbulent volume. Figure 2(a) shows a block diagram of tomographic measurements when using 3 guide stars.

In the optical layer oriented approach [OPTLO], a single wavefront sensor collects light from the ensemble of guide stars. For instance, pyramid wavefront sensors placed at the focal plane, and a common objective allow to superimpose pupil images coming from different guide stars. A beam splitter is employed to split light between the several wavefront sensing planes which are conjugated to different specific altitudes in the turbulent profile. Each of these wavefront sensing planes will see the layer it is conjugated to, but also superimposed wavefronts concerning the other layers. When the loop is closed, each detector is sensing the perturbation occurred in its own layer, being the perturbations of the other layers cancelled by the correction applied by the other DMs. This approach allows then to have independent ‘classical adaptive optics’ loops, linking each deformable mirror with its associated wavefront sensing plane (see Figure 1(b)). The measurement done by each detector is a linear combination of the tomographic measurements, in the ideal case where there is no noise.
When a wavefront sensing plane is placed at the pupil plane, the footprints of each of the stars overlap at the same position (i.e. the telescope pupil) while for the wavefront sensing planes situated at any other altitude, the measured phase will be the overlapping of the phase sampled by each star with the corresponding displacement given by $\vec{\phi} = h\vec{\phi}_g$, where $h = h_i - h_d$ is the difference between the layer altitude and the wavefront sensing plane altitude of conjugation and $\vec{\phi}_g$ is a vector parallel to the plane of the aperture which gives the position of the guide star (see Figures 1(b) and 2(b), where DM 1 is conjugated to the pupil plane and DM 2 is conjugated to a layer in altitude).

Due to the nature of OPTLO measurements, that makes that in close loop each detector senses essentially the wavefront perturbations occurring in its conjugated layer, the spatial and temporal sampling (at subaperture size, $\Delta t_i$ integration time) can be adjusted to the characteristics of the layer the detector is conjugated to. Due to the vertical distribution of the $C_n^2$ profile, that currently concentrates the most part of the turbulence in the layers near the ground, the upper layers will have a larger Fried parameter, and thus the subaperture size in a wavefront sensing plane conjugated to a high altitude layer can be larger than those employed in a wavefront sensing plane conjugate to the pupil plane, or in a wavefront sensor coupled to a guide star, where the spatial sampling is given by the global Fried parameter, always smaller than the Fried parameter corresponding to each layer.

As we mentioned before, optical layer oriented is a linear combination of tomographic measurements (outside of noise purposes) which is done optically. This combination of tomographic measurements could be also done numerically after the detection of the phase error by each of the wavefront sensors. In this hybrid technique which is called numerical layer oriented [NUMLO], we pass from having a set of measurements per guide star to having a set of measurements per layer. In the tomographic measurements, the spatial and temporal sampling is matched to the characteristics of the global atmosphere. In order to reproduce measurements of the same nature than OPTLO measurements, the same integration time and integration surface must be used. This is done numerically from tomographic measurements. If the coherence time of the upper layer is twice the global one, the integration time of the optical layer oriented measurements in this layer will be twice the integration time employed in the standard measurements and we will have to add two standard frames in order to reconstruct a single frame obtained by means of optical layer oriented measurements. The same applies to the integration surface.
In an ideal case, without any noise source, numerical and optical layer oriented measurements would be the same. In fact, the only difference between the measurements done by the two techniques is due to noise management. One can foresee that the NUMLO measurements have been created from tomographic measurements where a CCD is employed per guide star, thus increasing the read out noise when the number of guide stars is increased. On the other hand the flux is not divided in this technique, and thus increasing the number of DMs can be easily done without degrading the SNR.

<table>
<thead>
<tr>
<th>$N_{ph}^l(r)$</th>
<th>Number of photons per second per square meter per $\mu$m in a given layer $l$ of the atmosphere, depending on the GS geometry, and magnitude.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{bs}^l(r)$</td>
<td>Number of stars whose footprints contribute in $r$ in the layer $l$.</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Coherence time of the whole atmosphere.</td>
</tr>
<tr>
<td>$\tau_l$</td>
<td>Coherence time of layer $l$.</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Integration time employed for the whole atmosphere.</td>
</tr>
<tr>
<td>$\Delta t_l$</td>
<td>Integration time employed for layer $l$.</td>
</tr>
<tr>
<td>$d$</td>
<td>Sampling pitch for the wavefront measurements in NUMLO.</td>
</tr>
<tr>
<td>$d_l$</td>
<td>Sampling pitch for the wavefront measurements in OPTLO, layer $l$.</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Fried parameter describing the whole atmosphere.</td>
</tr>
<tr>
<td>$r_0,l$</td>
<td>Fried parameter associated to layer $l$.</td>
</tr>
<tr>
<td>$RON$</td>
<td>Read out noise of the CCD detector.</td>
</tr>
<tr>
<td>$\alpha_l$</td>
<td>Fraction of light arriving to the detector $l$ in the optical layer oriented, measurements due to the beam splitter.</td>
</tr>
<tr>
<td>$N_{bg}$</td>
<td>Number of background photons/sec/m$^{-2}$ corresponding to an area of 1 arcsec$^2$ on the sky.</td>
</tr>
<tr>
<td>QE</td>
<td>Quantum efficiency (it includes also optical transmission).</td>
</tr>
</tbody>
</table>

**Table 1.** List of parameters necessary to compute the SNR.

### 3. COMPUTATION OF SNR

In this Section, analytical formula for the numerical and optical layer oriented measurements SNR are developed. A large number of variables appearing, they are listed in Table 1. We assume that the sampling is adjusted to the local atmospheric conditions in each layer for the OPTLO, that is: $d_l = r_0,l$, $\Delta t = \tau_l$ and to the global atmospheric parameter for the NUMLO: $d = r_0$, $\Delta t = \tau$.

In order to compare fairly both techniques, we assume that the same wavefront sensor is used in both. This could be a pyramid wavefront sensor, whose behavior with respect to noise is similar to a Shack-Hartmann employing a quad-cell, as long as light is also divided in four pupil images. We will employ the expressions for the measurement error corresponding to a Shack-Hartmann wavefront sensor, which are better known. The expressions corresponding to signal photon noise, detector noise and background noise when employing a quad-cell are given by Rousset:

$$
\sigma_s^2 = \pi^2 \frac{1}{n_{ph}} \left( \frac{\theta d}{\lambda} \right)^2 \text{ rad}^2
$$

(1)

$$
\sigma_d^2 = 4\pi^2 \frac{RON n^2}{n_{ph}} \left( \frac{\theta d}{\lambda} \right)^2 \text{ rad}^2
$$

(2)

$$
\sigma_{bg}^2 = \frac{n_{bg}^2 \left( \frac{\theta d}{\lambda} \right)^2}{4} \text{ rad}^2
$$

(3)

where $d$ is the subaperture diameter, $n_{ph}$ is the number of photons per subaperture and exposure time, $RON$ is the rms number of noise electrons per pixel and per frame, $n_{bg}$ is the total number of photoelectrons from the sky background distributed over the 4 pixels in the quad-cell and $\theta$ is the angular size of the source.
image. For the sake of simplicity we will first consider the case in which subaperture images are limited by diffraction, and then $\theta_0 = \lambda/d$.

The ratio $1/\sigma$, where $\sigma$ is the root square of the sum of the error variances due to photonic, detector and background noise, can be taken as a signal to noise ratio in the flux detection, as it gives the usual expression for the SNR given by the ratio between the signal and square root of the sum of photon, detector and background noise:

$$\text{SNR} = \frac{n_{ph}}{\pi \sqrt{n_{ph} + 4\text{RDN}^2 + n_{bg}}} \quad (4)$$

Using the notation introduced in Table 1, and considering the optical layer oriented measurements, we can write for measurements in the layer $l$:

$$\text{SNR}^l_{OPT}(r) = \frac{c_l d^2 \gamma \text{QEN}^l_{ph}(r)}{\pi \sqrt{c_l d^2 \gamma \text{QEN}^l_{ph}(r) + 4\text{RDN}^2 + c_l d^2 \gamma \text{QEN}_{bg}} N^l_{gs}(r)} \quad (5)$$

where the signal term, as well as the photon noise and background noise terms are multiplied by $c_l$, a factor which determines the fraction of light that is sent to detector $l$, with $\sum c_l = 1$. We consider a field of 1 arcsec$^2$ per guide star in the computation of the background photons, and this term is then multiplied by the number of stars which contributes to the corresponding subaperture.

NUMLO measurements are created from tomographic measurements in which both spatial and temporal sampling were adjusted to match the global atmospheric characteristics. In reproducing the layer oriented measurements, the same sampling of layer oriented has to be used. If $d$ is the pixel size in the tomographic measurements and $d_l$ is the pixel size in the layer oriented measurements, the detector noise term has to be multiplied by a factor $(d/d_l)^2$ to account for the spatial binning required for layer oriented spatial sampling and by a factor $\gamma_l/\gamma$ that is the summation of sample times to account for the increased exposure time in layer oriented. Also, a factor $N^l_{gs}(r)$ accounts for the fact that tomographic measurements employ one detector per guide star. Taking all this in consideration, we can write the following expression for the NUMLO SNR:

$$\text{SNR}^l_{NUM}(r) = \frac{d^2 \gamma \text{QEN}^l_{ph}(r)}{\pi \sqrt{d^2 \gamma \text{QEN}^l_{ph}(r) + 4\text{RDN}^2 N^l_{gs}(r) + d^2 \gamma \text{QEN}_{bg} N^l_{gs}(r)}} \quad (6)$$

There is a potential gain in the OPTLO measurements where photons are additioned before detection, and thus, the resultant read out noise is reduced. However, there is a penalty in terms of flux for this type of measurements as we divide the flux between the different wavefront sensing planes. This penalty pops out by means of the factor $c_l$ which accounts for the fraction of light that is sent to this wavefront sensing plane, always smaller than one. These are two opposite effects that deserve a deeper study. Background noise behaves in the same way for both techniques except that the division of flux in the OPTLO measurements affects also background noise. Denoting $\gamma_l = \frac{d^2 \gamma_l}{d^2 l}$ we can write:

$$\text{SNR}^l_{OPT}(r) = \frac{c_l d^2 \gamma \text{QEN}^l_{ph}(r)}{\pi \sqrt{c_l d^2 \gamma \text{QEN}^l_{ph}(r) + 4\text{RDN}^2 + c_l d^2 \gamma \text{QEN}_{bg} N^l_{gs}(r)}} \quad (7)$$

$$\text{SNR}^l_{NUM}(r) = \frac{\gamma d^2 \gamma \text{QEN}^l_{ph}(r)}{\pi \sqrt{\gamma d^2 \gamma \text{QEN}^l_{ph}(r) + 4\text{RDN}^2 N^l_{gs}(r) + \gamma d^2 \gamma \text{QEN}_{bg} N^l_{gs}(r)}} \quad (8)$$

In the previous expressions (Equations 7 & 8), both the number of stars and the number of photons per $m^2$ per second depend on the position of the subaperture in the layer. For a wavefront sensing plane conjugated
to any layer but the telescope pupil, the footprints does not overlap (see Figure 2(b)), thus giving a density of photons and number of effective guide stars depending on the position of the subaperture in the detector. For a detector conjugated to the telescope pupil all the star footprints overlap, and thus the number of guide stars and the density of photons is constant. In the following, we will do the simplifying hypothesis of:

\[ N_{gs}'(r) = N_{gs} \], total number of guide stars,

\[ N_{ph}(r) = N_{ph} \], density of photons due to the contribution of all the guide stars.

As explained, this is perfectly valid in the pupil while in any other layer it means that we consider a portion of the layer where all the stars footprints overlap.

Equations 7 & 8 allow us to deduce the SNR ratio:

\[
\frac{SNR_{NUM}^j}{SNR_{OPT}^j} = \frac{1}{\alpha_l} \sqrt{\frac{\alpha_l \gamma r_0^2 \tau Q E(N_{ph} + N_{bg} N_{gs}) + 4 \text{RON}^2}{\gamma r_0^2 \tau Q E(N_{ph} + N_{bg} N_{gs}) + 4 \text{RON}^2 N_{gs} \gamma_l}}
\]  

(9)

In the simple case of having a detector without noise (RON=0.), \[ \frac{SNR_{NUM}^j}{SNR_{OPT}^j} = \frac{1}{\alpha_l} > 1 \] as \( 0 < \alpha_l < 1 \). The same result applies to very high flux cases where the readout noise can be neglected. In this situation there will be no advantage for the OPTLO measurements. In any other situation, the SNR ratio depends on a large number of factors: \( N_{gs}, N_{ph}, N_{bg}, \text{RON}, r_0, \tau, \gamma \) and \( \alpha_l \) as we will see in the following section.

Subaperture diffraction limited images occur when the subaperture diameter is smaller than the Fried diameter at the wavefront sensing wavelength. This is the case studied so far. In order to generalize expressions 7 and 8 to account for the case of subaperture images limited by seeing, the same process is followed, departing from Equations 3 and replacing the angular size of the image, \( \theta_s \), by \( \lambda / r_0(\lambda_{wfs}) \), corresponding to seeing limited images. This results in:

\[
SNR_{OPT}^j = \frac{\alpha_l \gamma d^2 \tau Q E_{ph}}{\pi \sqrt{\alpha_l \gamma d^2 \tau Q E_{ph} + 4 \text{RON}^2 + \alpha_l \gamma d^2 \tau Q E_{bg} N_{gs}}} \times \frac{r_0(\lambda_{wfs})}{d}
\]  

(10)

\[
SNR_{NUM}^j = \frac{\gamma d^2 \tau Q E_{ph}}{\pi \sqrt{\gamma d^2 \tau Q E_{ph} + 4 \text{RON}^2 N_{gs} \gamma_l + \gamma d^2 \tau Q E_{bg} N_{gs}}} \times \frac{r_0(\lambda_{wfs})}{d}
\]  

(11)

These are in fact the general expression. When wavefront sensing and correcting at the same wavelength, the subaperture size is set to match the size of the Fried parameter \( (d = r_0(\lambda_{wfs})) \). In this case, the multiplying factor in Equations 10 & 11 is equal to one, and we find again the expressions 7 & 8.

4. STUDY CASES

We have chosen three scenarios which are representative of different observing conditions of this type of systems:

- **Case 1**: wfs and correction at the K band (2.20 \( \mu \)m).
- **Case 2**: wfs and correction at the R band (0.64 \( \mu \)m).
- **Case 3**: wfs at the R band and correction at the K band.

The SNR depending on a large number of parameters, we are obliged to choose the value of some of them which are given in Table 2. This Table show the wavefront sensing and correction wavelength corresponding to each case. The bandwidth is equal for the three cases, \( \Delta \lambda = 0.4 \mu \text{m} \). As a first approximation, the quantum efficiency (QE) is also equal for the three cases, even if it could be smaller for infrared detectors. We remind that in this work, the QE accounts also for optical transmission, as stated in Table 1. Fried parameter corresponds to a seeing of 0.7 arcsec in the visible. RON values correspond to the state-of-art of visible and infrared detectors. \( m_{bg} \) accounts for the sky background magnitude at the wavefront sensing wavelength. The integration time is taken to be constant and equal to 5.0 msec. Unless specified otherwise the number of guide stars is 10.
<table>
<thead>
<tr>
<th></th>
<th>$\lambda_m (\mu m)$</th>
<th>$\lambda_{wfs} (\mu m)$</th>
<th>$\Delta \lambda (\mu m)$</th>
<th>QE</th>
<th>$v_{0,wfs}(m)$</th>
<th>$v_{0,wm}(m)$</th>
<th>RON</th>
<th>$m_{bg}$</th>
<th>$\tau$(msec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>2.20</td>
<td>2.20</td>
<td>0.4</td>
<td>0.4</td>
<td>0.80</td>
<td>0.89</td>
<td>18.0</td>
<td>13.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.64</td>
<td>0.64</td>
<td>0.4</td>
<td>0.4</td>
<td>0.20</td>
<td>0.20</td>
<td>3.0</td>
<td>20.5</td>
<td>5.0</td>
</tr>
<tr>
<td>Case 3</td>
<td>2.20</td>
<td>0.64</td>
<td>0.4</td>
<td>0.4</td>
<td>0.20</td>
<td>0.89</td>
<td>3.0</td>
<td>20.5</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Table 2. Basic parameters used for the SNR estimation.

Figure 3. Numerical and optical SNR when measuring and correcting in the K band (case 1) as a function of the equivalent magnitude in the K band when employing 10 guide stars. Continuous lines correspond to numerical measurements and dashed lines correspond to optical measurements. Diamond symbols are used for $\gamma = 10$, while no symbols are used for $\gamma = 1$. Parameters are given in Table 2.

In the following subsections, we will consider two conjugate planes ($\alpha_1 = \alpha_2 = 0.5$) and study the dependence of optical and numerical SNR with the amount of flux (subsection 4.1) and we will investigate the gain in magnitude as a function of the number of guide stars (subsection 4.2).

4.1. SNR versus equivalent magnitude

We represent the SNRs as a function of the equivalent magnitude in the wavefront sensing band. This equivalent magnitude is defined as the magnitude of a fictive star with a number of photons per square meter per second equal to $N_{ph}$, which is the total photon density given by the whole set of guide stars. The flux at wavelength $\lambda$ in units of $W m^{-2} \mu m^{-1}$ is given by $F_0(\lambda) = 10^{-0.4M_{\lambda}-Z_{BAND}}$, where $Z_{BAND}$ is the zero point given in Table 3.

The number of photons per square meter per second per $\mu m$ is then found dividing the flux by the energy of the photon at the corresponding wavelength.

<table>
<thead>
<tr>
<th>Band</th>
<th>$\lambda_0 (\mu m)$</th>
<th>$Z_{BAND}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>0.64</td>
<td>7.6408</td>
</tr>
<tr>
<td>K</td>
<td>2.20</td>
<td>9.4367</td>
</tr>
</tbody>
</table>

Table 3. Magnitude zero points employed in computing the equivalent magnitude corresponding to a number of photons per square meter and per second equal to $N_{ph}$.
Figure 4. SNR versus equivalent magnitude at the measuring wavelength. Continuous lines stands for numerical and dashed lines for optical. Curves corresponding to two values of \( \gamma \) are shown: \( \gamma = 1 \), without symbols ; \( \gamma = 10 \), diamond symbols. For the three cases we have assumed \( N_p = 10 \).

Figure 3 shows the numerical and optical measurements SNR as a function of the equivalent magnitude and for different values of \( \gamma \) (1 and 10) in the case when wavefront sensing and correction in the K band (case 1). This Figure allows to see the main features of signal to noise ratio behavior. At small equivalent magnitude, the SNR is dominated by the photon noise, and both detector and background noise are negligible. In this case, the NUMLO SNR is larger than the OPTLO by a factor \( 1/\sqrt{\alpha_1} \) as explained in Section 3, due to the flux division between the detectors done in the OPTLO configuration. The SNR is proportional to the number of photons in this region. Around equivalent magnitude \( m_k = 6 \) the behavior is inverted and OPTLO SNR is larger. In this region, we are dominated by detector noise and the SNR is proportional to the square root of the number of photons. For the values of RON considered (see Table 2) which corresponds to infrared detectors state-of-art, the detector noise term is larger than the contribution of background noise. Furthermore, detector noise term is more limiting in the NUMLO configuration and depends on the value of \( \gamma \), this is why when increasing this parameter, the separation between OPTLO and NUMLO SNR also grows. In other words, optical layer oriented is particularly sensitive in high altitude layers where spatial and temporal sampling are optimized.

Notice that we have employed a constant exposure time. In a real system, when the SNR is too low to allow the reconstruction of the phase, the exposure time is increased in order to increase the signal. In that case, the detector noise per frame will be the same, but the background noise will increase and become dominant. In that case the difference between both measurements in terms of signal to noise ratio will decrease.

This curve allows us to understand the asymptotic behaviors of the system (at very high and very low fluxes) but the operational domain of most systems is probably limited to a small SNR range. For very high SNR the performance of the system is limited by other aspects (for instance the number of actuators in the DM) and for very small SNR the measurements are too noisy to allow the reconstruction of the phase.

Figures 4 (a), (b) and (c) show the SNR versus the equivalent magnitude in each of the wavefront sensing bands, for the three cases defined in Section 4. The SNR is limited to the range 0.1 to 10 and the number of guide stars is 10. Figure 4(a) is in fact a zoom of the center of Figure 3. The case of \( \gamma = 1 \) rather corresponds to the ground layer, where most of the turbulence is concentrated, and thereby local Fried parameter is close to the global one. While \( \gamma = 10 \) could correspond to a high altitude layer with a local Fried parameter approximately three times larger than the global one, and an exposure time approximately 1.3 times larger than the global one. Increasing \( \gamma \) translates into increasing the SNR in both techniques as the signal itself is increased. However, in the NUMLO approach, this effect is balanced by the detector noise term which increases also with \( \gamma \). Case 1 is the most favorable for OPTLO measurements (see Figure 4(a)). For both values of \( \gamma \), the OPTLO SNR is larger than NUMLO SNR in the shown domain. In this case, NUMLO measurements are penalized by the large RON of infrared detectors. In case 2, shown in Figure 4(b), OPTLO measurements have larger SNR than NUMLO measurements for equivalent magnitudes larger than \( m_R = 11.5 \) (if we consider that all the stars...
have the same magnitude, this corresponds to individual star magnitudes of $m_R = 13.5$). Both in case 1 and 2, the wavefront sensing wavelength is the same as the correction wavelength, and we have considered that subaperture images are diffraction limited. In case 3, the wavefront sensing is done in the R band while the correction is done in the K band, and thus subaperture images are seeing limited. This means a loss of SNR of a factor $r_0(\lambda_{ws})/r_0(\lambda_{cor})$ with respect to the case then wavefront sensing and correcting in the K band, but the detector noise is less limiting here as we use detectors in the visible range with smaller RON. The background noise is also smaller when wavefront sensing in the visible with respect to the case when wavefront sensing in the infrared. The point at which NUMLO and OPTLO SNR crosses is now $m_R \approx 14$ (see Figure 4(c)). Star configurations with smaller equivalent magnitude will have better SNR when doing NUMLO measurements, while with larger equivalent magnitude, the SNR will be better for OPTLO measurements (as before, individual star magnitude would correspond to two magnitudes more, that is $m_R \approx 16$ if stars are of the same magnitude).

4.2. Magnitude gain

Up to now, we have not considered the influence of the number of guide stars in the SNR comparison. Figures 5(a), (b) and (c) show the gain in magnitude ($\Delta m = m_{OPTLO} - m_{NUMLO}$) for the three study cases and for three different values of $SNR = 0.5, 1.0, 2.0$ as a function of the number of guide stars. This gain in magnitude is computed as the difference in equivalent magnitude needed for achieving a fixed SNR with the two techniques. An example of its definition is given in Figure 4(a), where we show the magnitude gain for two values of $\gamma$ (1 and 10) when the SNR is equal to 0.5. The numerical values corresponding to this example are $\Delta m \approx 1.5$ for $\gamma = 10$ and $\Delta m \approx 0.5$ for $\gamma = 1$. It has been defined so that a negative gain is favorable for the numerical approach while a positive gain is favorable for the optical approach. The gain in magnitude depends strongly on the value of $\gamma$, that is to say, on the local atmospheric conditions of the layer with respect to the global atmospheric conditions as can be seen in Figure 5. Larger values of $\gamma$ means an increase in terms of magnitude gain, as well as increasing the number of guide stars increase also the magnitude gain.

Although optical layer oriented was proposed to employ and profit of a large number of guide stars, the number of guide stars that can be used is finally limited by the free space in the focal plane, and by the number of guide stars available in the FOV. Three to five guide stars will be reasonable for the north galactic pole (NGP) while at the equator, the number of available guide stars could be about ten.

In the two first cases (Figures 5(a) and (b)) it can be seen that the OPTLO configuration can provide a magnitude gain even with a small number of guide stars (3-4), provided $\gamma > 1$. In the third case, when the wavefront sensing is done at the R band and correction at the K band, Figure 4(c) shows that the point where the NUMLO and OPTLO SNR curves cross is displaced towards larger magnitudes, thus for the chosen values of $SNR = 0.5, 1.0, 2.0$, the gain is more modest than in the other two cases, as can be seen in Figure 5(c).

The magnitude gain is relatively independent of SNR values in case 1, while in case 2 and 3, for the same value of $\gamma$, the gain increases when the SNR decreases. This is due to the fact that the magnitude gain depends on the point at which both OPTLO and NUMLO SNR curves crosses and this point is determined by the weight of the detector noise term (always more limiting than the background noise term in this study) with respect to the signal. In case 3 for instance, the spatial sampling is reduced by the fact that we are measuring in the R band to correct in the K band. In this case, the amount of flux per subaperture is larger and the detector noise is less limiting as explained also in Subsection 4.1.

5. DISCUSSION

Apart from SNR considerations, NUMLO or OPTLO options have each one its own advantages and drawbacks. The optical approach will allow to do a better spatial and temporal sampling of the upper layers. We mentioned before that the Fried parameter is larger in the upper atmospheric layer thus inducing a possibility to increase the spatial sampling, increasing the signal to noise ratio and reducing also the data volume obtained from the wavefront sensors. Optical layer oriented was in fact proposed with the idea of employing a large number of guide stars, as the complexity of the system does not increase in this case with the number of guide stars (although the number of guide stars holds restrained due to the requirements for locating the corresponding pyramids wavefront sensors in the focal plane), but with the number of deformable mirrors. In the case of
Figure 5. Gain in magnitude as a function of the number of guide stars, for $\gamma = 1$ and $\gamma = 10$, and several values of SNR. Dashed lines correspond to SNR = 20, continuous lines to SNR = 1.0 and dashed-dotted lines to SNR = 0.5.

numerical layer oriented, the increase of guide stars is done at the expense of increasing also the detector noise, as a CCD is employed per guide star in this case. The hardware is also more complicated in this case, but this could be an advantage also as in the NUMLO system the degradation of one CCD is not essential for the functioning of the system, as long as we are working with a large number of guide stars. This larger number of CCDs in the numerical case entrains also a problem of cabling and optical relay, and a larger weight in the focal plane, which is convenient to be reduced to the minimum value. However this strategy could allow to weight differently the wavefront coming from guide stars of different magnitude. The numerical approach is also more versatile in the sense that another DM can be added directly, whether in the optical approach if doing it directly, photons would have to be shared between a new wavefront sensing plane. In the optical approach, one potential disadvantage is the need for large optics as each wavefront sensing plane would see the entire reference filed of view, but recently several solutions are proposed to this problem.12

Considering only SNR the analytical expressions developed here allow us to establish a comparison between the two techniques. It is seen that OPTLO measurements are favored when increasing the number of guide stars, and at low flux regimes. This option seems to be more efficient when wavefront sensing and correcting in the R band or in the K band. We recall that when the SNR is too small in a real system, the exposure time is increased, while it has been kept constant in this work due to the large number of parameters. Increasing the exposure time, increases the background noise with respect to the detector noise and thus the difference between NUMLO and OPTLO will be attenuated.

NUMLO measurements are penalized by the detector noise term. Case 3, when measuring in the R band and correcting in the K band is the most favorable for this type of measurements, as the amount of signal is increased by employing the spatial sampling given by the Fried parameter at the correction wavelength, at the time that employing visible detector with smaller $RON$.

ACKNOWLEDGMENTS

This research has been benefited from the support of the European Commission RTN program: ‘Adaptive Optics for the Extremely Large Telescopes’, contract n. HPRN-CT-2000-00147.

REFERENCES


